

Ptolemy's Observation

nine etudes in extended 7-limit just intonation
for four or more string instruments

Taylor Brook
2016

Instrumentation:

4 or more stringed instruments in any combination of cellos, violas, and violins.

Etude I, VII, and IX are possible with only 3 instruments, while all others require at least 4.

About *Ptolemy's Observation*

Ptolemy's Observation was written for Mira Benjamin as a set of studies for teaching basic just intonation performance on stringed instruments.

The name of these studies refers to the Greco-Egyptian mathematician and philosopher from the second century, Ptolemy, who extended the Pythagorean conception of musical harmony from a 5-limit to a 7-limit just intonation in his influential treatise on music entitled *Harmonics*.

The score may be played by any number of string players beyond four, and in any combination of, cellos, violas, and violins. Each study uses a drone with two or three soloists playing on top. In performance, the musicians may take turns playing the solo parts while the remainder create the drone. The movements often repeat at different tempi, allowing for roles to be passed around and switched between players as they like.

Just Intonation - a basic primer

Just intonation is a tuning system that appeals to the acoustic consonance of musical intervals through whole-number ratios between frequencies. The logic of this lies in the coincidence of the waveforms resulting in a harmonic relationship. If two notes are of the same pitch, their wave forms will coincide with a 1/1 relationship. If we then double the frequency of the first note we then have an octave, which has a 2/1 relationship. If we then continue up the harmonic series and double the frequency once more, we now have a 3/1 relationship, which is a just major fifth plus an octave, which we could simplify to a 3/2 to make a perfect fifth. From here we can continue, reaching the 5/4 (just major third), 6/5 (just minor third) and so on. As the numbers in the ratios increase the intervals become less consonant, each having their own identity.

In the context of just intonation a “limit” will refer to the largest prime number used in the ratios that describe the intervals.

Basic 5-limit just intonation in relation to 12-tone equal temperament:

1/1	9/8	6/5	5/4	4/3	3/2	5/3	8/5	15/8	1/1
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0c	+4c	+16c	-14c	-2c	+2c	+14c	-16c	-12c	0c
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for more information:

https://en.wikipedia.org/wiki/Five-limit_tuning

Basic 7-limit just intonation in relation to 12-tone equal temperament:

1/1	-9/8	-8/7	-7/6	-6/5	-5/4	-9/7	-4/3	-7/5	-10/7	-3/2	-8/5	-5/3	-12/7	-7/4	-15/8	-1/1
-----	------	------	------	------	------	------	------	------	-------	------	------	------	-------	------	-------	------

0c	+4c	+31c	-33c	+16c	-14c	+35c	-2c	-17c	+17c	+2c	+18c	-16c	+33c	-31c	-14c	0c
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for more information:

https://en.wikipedia.org/wiki/7-limit_tuning

Instrumentation:

Each movement contains a “drone” staff as well as two or three “solo” staves. With groups of 4-8 players, I would suggest that single instruments perform the “solo” staves while the remainder of the musicians all play the drone part. The performers may then choose to alternate roles upon repeats or between movements as they desire. If the ensemble is larger than eight players, the “soloist” staves could be doubled or tripled and so on as the size of the ensemble requires.

Double bass players may be included as well, as all the harmonics are played on the A, D, and G strings, but they will need to change octaves as their instrument demands.

Microtonal Notation:

The microtonal notation contains three key pieces of information. First, above the note there is a fraction that shows the identity of the interval. These ratios are similar to roman numeral harmonic analysis, providing information so that the players may understand the harmony and their role in that harmony.

Secondly, also above the notes, there is cent notation which shows the deviation in hundredths of a semitone from the nearest 12-tone equal temperament note. This notation is useful in combination with a tuner that shows cent deviations. For practicing or during performance, one can check their pitch using a tuner by referring to the cent notation.

Lastly, microtonal accidentals are used, providing precision to the nearest 12th of a tone. This notation is more approximate and may provide an intuitive sense of where the note is on the fingerboard.

♭ - ♯ approximately 1/4 tone flat or sharp (50 cents)

↓ - ↑ approximately 1/6 tone flat or sharp (33 cents)

♭ - ♭ - ♭ - ♭ - ♭ - ♭ - ♭ - ♭ approximately 1/12 tone flat or sharp (17 cents)

The image shows a musical score for three parts: Solo 1, Solo 2, and Drone. The score is written in treble clef for the solo parts and bass clef for the drone part. The drone part consists of a series of chords, each with a microtonal accidental. The solo parts consist of a series of notes, each with a microtonal accidental. The notes in Solo 1 and Solo 2 are connected by a slur. The drone part is a series of chords, each with a microtonal accidental. The notes in Solo 1 and Solo 2 are connected by a slur. The drone part is a series of chords, each with a microtonal accidental.

Measure	Interval Ratio	Cent Deviation	Microtonal Accidental
1	5/4	-12c	♭
2	7/5	-10c	♭
3	15/8	-12c	♭
4	5/3	-14c	♭
5	5/4	-12c	♭
6	15/8	-12c	♭
7	5/4	-14c	♭

Tuning the open strings:

The instruments should be tuned in the following way to get the highest level of precision.

Two tunings are possible for the violins. The second tuning allows for easy access to a recurring pitch in the etudes but breaks the chain of perfect 5ths in the interval between the A and E strings. I would encourage the violinists to explore both tunings and use whichever they prefer.

1/1 3/2 9/8 27/16
0c +2c +4c +6c

violin
option 1

1/1 3/2 9/8 5/3
0c +2c +4c -16c

violin
option 2

4/3 1/1 3/2 9/8
-2c 0c +2c +4c

viola

4/3 1/1 3/2 9/8
-2c 0c +2c +4c

cello

Harmonics:

Natural harmonics are notated as the resultant pitch with a small circle above the notehead combined with a roman number that indicates the string and a small number indicating the harmonic that should be played. Diamond noteheads, showing one option of where to place the finger on the string are provided in the first two studies and omitted thereafter.

IV² IV³ IV⁴ IV⁵ IV⁶ IV⁷

violin

II² II³ II⁴ II⁵ II⁶ II⁷

cello/
vla (8vb)

Ptolemy's Observation

Just Intonation Etudes for Strings

Taylor Brook

I - Major Thirds

$\bullet = 60$
 $\text{♩} = 60$
 $\circ = 60$

$15/8$ -12c
 $5/4$ -12c
 $15/8$ -10c
 $7/5$ -10c
 $5/4$ -14c
 $5/3$ -14c

$8^{va} - \uparrow$
 $8^{va} - \uparrow$

Soloist 1
 Soloist 2
 Drone

mp
 mp
 mp

$3/2 (+2c)$
 $2/1$
 $9/8 (+4c)$
 $3/2 (+2c)$

$5/4$ -12c
 $7/5$ -10c
 $15/8$ -12c
 $5/3$ -14c
 $5/4$ -12c
 $15/8$ -12c
 $5/4$ -14c

8
 $8^{va} - \uparrow$

$5/4$ -12c
 $15/8$ -12c
 $5/3$ -16c
 $3/2$ +2c
 $1/1$ +2c
 $5/3$ -14c
 $3/2$ +4c
 $9/8$ +4c
 $1/1$ 0c

Solo 1
 Solo 2
 Drone

II - Just Minor Seventh

♩ = 60

♩ = 60

○ = 60

$\frac{7}{4}$
IV⁷ -31c

$\frac{7}{6}$
-31c

$\frac{3}{2}$
+2c
IV⁶

$\frac{5}{4}$
-14c
IV⁵

8^{va}

15

Solo 1

Solo 2

Solo 3

Drone

mp

$\frac{7}{4}$ -31c

$\frac{7}{6}$ -31c

$\frac{7}{4}$ -31c

$\frac{5}{4}$ -14c

$\frac{3}{2}$ (+2c)
2/1

$\frac{9}{8}$ (+4c)
 $\frac{3}{2}$ (+2c)

21

Solo 1

Solo 2

Solo 3

Drone

III⁷ $\frac{7}{4}$ -29c
(8^{va})₋₁

III⁶ $\frac{9}{8}$ +4c

III⁵ $\frac{15}{8}$ -12c

IV⁵ $\frac{5}{4}$ -14c

$\frac{7}{4}$ -31c

$\frac{7}{4}$ -31c

$\frac{7}{6}$ -31c

$\frac{7}{4}$ -31c

$\frac{7}{6}$ -31c

$\frac{7}{4}$ -31c

$\frac{5}{3}$ -14c

$\frac{7}{4}$ -29c

$\frac{9}{8}$ +4c

$\frac{15}{8}$ -12c

$\frac{3}{2}$ +2c

1/1 +2c

$\frac{5}{3}$ -14c

$\frac{5}{4}$ -14c

III - Mixture

♩ = 60
♩ = 60
○ = 60

27

Solo 1: $IV^5 \frac{5}{4} -14c$, $IV^7 \frac{7}{4} -31c$, $III^5 \frac{5}{4} -12c$, $III^7 \frac{7}{4} -29c$, $IV^5 \frac{5}{4} -14c$, $III^3 \frac{9}{8} +4c$

Solo 2: $5/4 -14c$, $7/4 -31c$, $15/8 -12c$, $5/3 -14c$, $7/6 -31c$, $3/2 +4c$

Solo 3: $15/8 -10c$, $7/4 -29c$

Drone: $3/2 (+2c)$, $2/1$, $9/8 (+4c)$, $3/2 (+2c)$

mp

33

Solo 1: $II^5 \frac{5}{4} -10c$, $III^3 \frac{3}{2} +6c$, $II^7 \frac{7}{4} -27c$, $IV^5 \frac{5}{3} -14c$, $III^7 \frac{7}{4} -29c$, $III^5 \frac{5}{4} -12c$, $IV^7 \frac{7}{4} -31c$

Solo 2: $1/1 +4c$, $3/2 +6c$, $15/8 -8c$, $7/5 -8c$, $5/3 -14c$, $7/6 -31c$, $5/4 -14c$

Solo 3: $7/6 -29c$, $5/4 -10c$, $15/8 -10c$, $5/4 -14c$, $7/4 -29c$, $15/8 -12c$

Drone: $1/1 +4c$, $3/2 +6c$, $15/8 -8c$, $7/5 -8c$, $5/3 -14c$, $7/6 -31c$, $5/4 -14c$

IV - Triads 1

Intervals shown are melodic, in reference to a tonic G 1st system, harmonic in the second system.

♩ = 60

♩ = 60

40

Solo 1

Solo 2

Solo 3

Vln. Drone

Drone Vla./Vcl.

9/8 +4c

5/4 -14c

7/5 -10c

3/2 +2c

3/2 of 9/8 +6c

15/8 -12c

IV²

II

IV⁵ 5/4 -14c

II⁵ *va-₁*

III²

II³

III⁵

IV⁴

III²

I

III⁵ 5/4 -14c

I⁵ 7/5 -10c

II² 4/3 -2c

I³ 3/2 of 9/8 +6c

II⁵

III⁴

48

Solo 1

Solo 2

Solo 3

Vln. Drone

Drone Vla./Vcl.

1/1 +2c

6/5 +2c

1/1 +4c

6/5 0c

6/5 +4c

6/5 +2c

6/5 +4c

3/2 +2c

3/2 -14c

1/1 -14c

5/4 -8c

3/2 -14c

3/2 -10c

1/1 -14c

3/2 -10c

5/4 -14c

5/4 -12c

3/2 -12c

3/2 +6c

1/1 -16c

1/1 -12c

3/2 -12c

1/1 -12c

1/1 0c

V - Triads 2

Intervals shown are melodic, in reference to a tonic D 1st system, harmonic in the second system.

♩ = 60
♩ = 60

56

1/1 +2c 8/7 -16c 7/4 -29c

Solo 1

Solo 2

Solo 3

Vln. Drone

Drone Vla./Vcl.

7/6 -31c 4/3 0c

3/2 +4c 5/3 -14c

1/1 +2c 8/7 -16c IV⁷ 7/6 -31c IV² 4/3 0c 3/2 +4c IV⁵ 5/3 -14c III⁷ 7/4 -29c III² 1/1 +2c

II² 1/1 +2c IV⁵ 8/7 -16c III⁷ 7/6 -31c III⁴ 4/3 0c I² 3/2 +4c III⁵ 5/3 -14c II⁷ 7/4 -29c II⁵ 1/1 +2c

64

1/1 -16c 3/2 -29c 3/2 +2c 7/6 -29c 6/5 +2c 7/6 -29c 1/1 -29c 3/2 -29c 1/1 +2c

Solo 1

Solo 2

Solo 3

Vln. Drone

Drone Vla./Vcl.

6/5 0c 1/1 -31c 1/1 0c 3/2 +6c 7/5 -31c 3/2 +6c 9/7 +6c 1/1 -31c 7/6 -31c

3/2 -14c 9/7 +4c 5/4 -14c 1/1 +4c 1/1 -14c 1/1 +4c 9/7 +4c 3/2 +4c

VI - Triads 3

Intervals shown are melodic, in reference to a tonic A 1st system, harmonic in the second system.

♩ = 60

♩ = 60

73

Solo 1

Solo 2

Solo 3

Vln. Drone

Drone Vla./Vcl.

1/1 0c

8/7 -14c

9/8 +8c

5/4 -10c

3/2 +6c

5/3 -12c

1/1 +4c

III⁵ 8/7 -14c

I³ 9/8 +8c

II⁵ 5/4 -10c

4/3 +2c

4/3 of 8/7 -16c

3/2 +6c

II⁵ 5/3 -12c

IV⁴ 16/9 0c

1/1 +4c

III⁵ 8/7 -14c

9/8 +8c

I⁵ 5/4 -10c

4/3 +2c

4/3 of 8/7 -16c

I³ 3/2 +6c

III⁵ 5/3 -12c

III⁴ 16/9 0c

82

Solo 1

Solo 2

Solo 3

Vln. Drone

Drone Vla./Vcl.

5/4 -14c

1/1 +4c

1/1 -14c

4/3 +4c

1/1 -12c

4/3 +4c

3/2 -14c

5/4 -12c

7/5 0c

4/3 +4c

1/1 -14c

1/1 +4c

3/2 +2c

9/8 +8c

9/8 -10c

3/2 +8c

3/2 -10c

3/2 +8c

9/5 +2c

1/1 +2c

1/1 -10c

3/2 +8c

9/8 -10c

9/8 +8c

5/4 -10c

1/1 0c

3/2 +6c

3/2 -12c

1/1 +6c

1/1 +6c

1/1 -16c

3/2 +4c

6/5 +6c

1/1 +6c

4/3 -16c

3/2 +6c

IV⁴ 1/1 0c

1/1 +4c

III⁵ 1/1 -14c

1/1 +6c

II⁵ 1/1 -12c

1/1 +6c

1/1 -16c

1/1 +2c

II⁵ 1/1 -10c

1/1 +6c

III⁵ 1/1 -14c

1/1 +4c

III⁴ 1/1 0c

1/1 +4c

III⁵ 1/1 -14c

I³ 1/1 +6c

III⁵ 1/1 -12c

I³ 1/1 +6c

IV⁵ 1/1 -16c

1/1 +2c

I⁵ 1/1 -10c

I³ 1/1 +6c

III⁵ 1/1 -14c

1/1 +4c

VII - Canon 1 - harmonics versions

Entrances alternate between whole note and breve (double whole note).

♩ = 60
 ♩ = 60
 ♩ = 60

95

6/5 +2c
 1/1 -14c
 3/2 -12c
 6/5 +2c
 3/2 -12c
 9/5 +4c
 6/5 +2c
 4/3 -16c
 3/2 -12c
 9/5 +4c
 1/1 -14c

Canon

vln: III² IV⁵ III⁵ III² III⁵ III² III² IV⁵ III⁵ III² IV⁵
 vla/vcl: II⁴ III⁵ II⁵ II⁴ II³ I⁴ II⁴ IV⁵ II³ I⁴ III⁵

VII - Canon 1 - fingered with drone

♩ = 60
 ♩ = 60
 ♩ = 60

103

6/5 +2c
 1/1 -14c
 3/2 -12c
 6/5 +2c
 3/2 -12c
 9/5 +4c

Solo 1

Solo 2

Solo 3

Vln. Drone

Drone Vla./Vcl.

1/1 -14c
 IV⁵ 1/1 -14c

6/5 +2c
 1/1 -14c
 3/2 -12c
 6/5 +2c
 1/1 -14c

6/5 +2c
 1/1 -14c
 3/2 -12c
 6/5 +2c
 3/2 -12c

6/5 +2c
 1/1 -14c

110

6/5 +2c
 4/3 -16c
 3/2 -12c
 9/5 +4c
 1/1 -14c
 8/5 +0c
 9/5 +4c
 3/2 -12c

Solo 1

Solo 2

Solo 3

Vln. Drone

Drone Vla./Vcl.

9/5 +4c
 6/5 +2c
 4/3 -16c
 3/2 -12c
 9/5 +4c
 1/1 -14c
 9/8 -10c
 6/5 +2c

3/2 -12c
 6/5 +2c
 3/2 -12c
 9/5 +4c
 6/5 +2c
 4/3 -16c
 3/2 -12c
 9/5 +4c
 1/1 -14c

VIII - Canon 2

♩ = 60

♩ = 60

♩ = 60

117

Solo 1

Solo 2

Solo 3

Vln. Drone

Drone Vla./Vcl.

9/7 +6c

10/7 -14c

12/7 +2c

3/2 -29c

12/7 +2c

1/1 -31c

12/7 +2c

12/7 +2c

1/1 -31c

8/7 0c

9/7 +6c

10/7 -14c

1/1 -31c

1/1 III⁷ -31c

125

Solo 1

Solo 2

Solo 3

Vln. Drone

Drone Vla./Vcl.

1/1 -31c

8/7 0c

12/7 +2c

3/2 -29c

9/7 +6c

10/7 -14c

3/2 -29c

9/7 +6c

12/7 +2c

10/7 -14c

9/7 +6c

1/1 -31c

IX - Canon 3 - harmonics

Entrances at whole note.

♩ = 60
 ♩ = 60
 ♩ = 60

1/1 9/8 5/4 5/3 15/8 1/1 9/8 5/4 9/8 1/1 1/1 9/8 5/4 1/1 4/3 1/1 3/2 1/1
 0c +4c -14c -16c -12c 0c +4c -14c +4c 0c 0c +4c -14c 0c -2c 0c +2c 0c

Canon

vln: IV⁴ III³ IV⁵ III⁵ IV⁴ III³ IV⁵ III³ IV⁴ III³ IV⁵ III³ IV⁴ III³ IV⁵ IV² III² IV²
 vla/vcl: III⁴ II³ III⁵ IV⁵ II⁵ III⁴ II³ III⁵ II³ III⁴ II³ III⁵ III⁴ IV⁴ III⁴ II⁴ III⁴

IX - Canon 3 - fingered with drone

♩ = 60
 ♩ = 60
 ♩ = 60

146

1/1 9/8 5/4 5/3 15/8 1/1 9/8 5/4 9/8
 0c +4c -14c -16c -12c 0c +4c -14c +4c

Solo 1

Solo 2

Solo 3

Drone

153

1/1 1/1 9/8 5/4 1/1 4/3 1/1 3/2 1/1
 0c 0c +4c -14c 0c -2c 0c +2c 0c

Solo 1

Solo 2

Solo 3

Drone